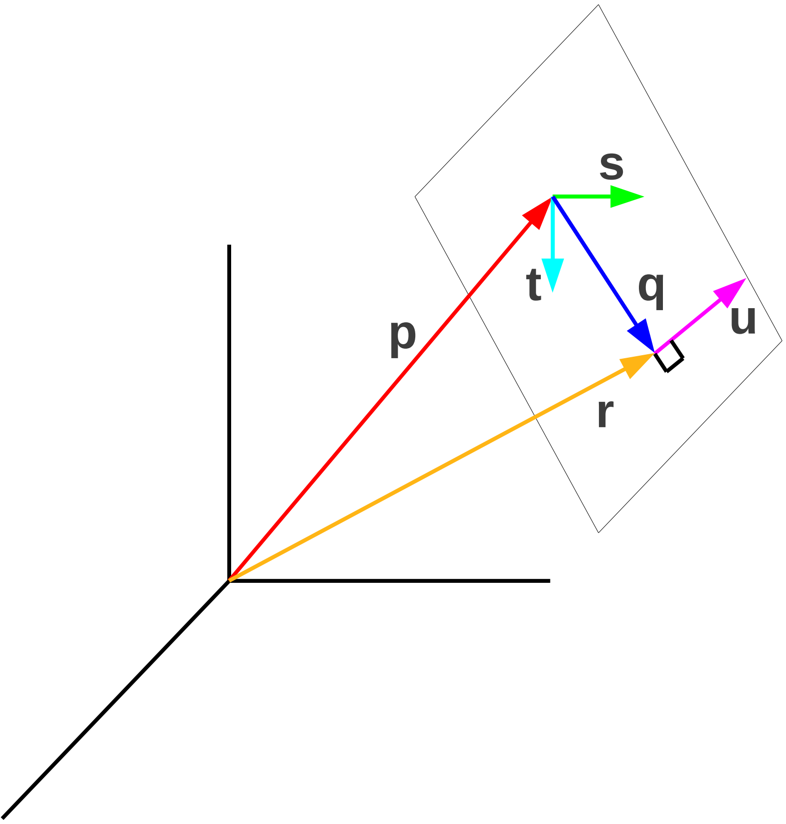
Find the set of vectors that lie in the planes that are parallel to 3*x* + 4*y* + 5*z* = 6 and are located 2 units away from it.



First, find a vector from the origin to the plane. Call it **p**. Any point in the plane will do, for instance (2, 0, 0).

$$p = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

Next, find two linearly independent vectors **s** and **t** that lie in the plane. To satisfy this requirement, the dot product of both **s** and **t** with the plane's perpendicular vector ⟨3, 4, 5⟩ must be 0.

I decided to use **s** = ⟨2, 1, -2⟩ and **t** = ⟨1, -2, 1⟩.

A linear combination of **s** and **t**, **q** = a**s** + b**t** (where a and b are constants) can describe any directional vector in the plane.

$$q = a\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} + b\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

The vector sum **r** = **p** + **q** (see illustration) is the position vector of any point in the plane (the exact point described depends on the chosen values of a and b).

To find points 2 units above and below the plane, we must find a vector **u** that is perpendicular to the plane and has a magnitude = 2. We therefore multiply the perpendicular vector ⟨3, 4, 5⟩ by a coefficient *k* and set the magnitude of the result equal to 2.

$$|\langle 3k, 4k, 5k\rangle| = \sqrt{(3k)^2 + (4k)^2 + (5k)^2} = \sqrt{50k^2} = 2$$

$$\Rightarrow k = \frac{\sqrt{2}}{5}$$

Hence, $u = \left\langle \frac{3\sqrt{2}}{5}, \frac{4\sqrt{2}}{5}, \frac{5\sqrt{2}}{5} \right\rangle$

The vector sum of a the position vector of a point in the plane **r** with **u** is the sought-after vector **v**. It is given by

$$\mathbf{v} = \mathbf{r} + \mathbf{u} = \mathbf{p} + \mathbf{q} + \mathbf{u} = \boxed{\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + a\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} + b\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3\sqrt{2}/5 \\ 4\sqrt{2}/5 \\ 5\sqrt{2}/5 \end{pmatrix}}$$